DISCUSSION ON DR. D. GABOR'S PAPER "COMMUNICATION THEORY AND PHYSICS".

PROF. B. VAN DER POL.

The expression synthesizing a function $f(t)$ (which contains no frequency components higher than critical angular frequency ω_o) from its equally spaced discrete values $f(\frac{\pi n}{\omega_o})$, where $n = \ldots -2,-1,0,1,2\ldots$ is

$$f(t) = \sum_{-\infty}^{+\infty} f(\frac{\pi n}{\omega_o}) . \frac{\sin(\omega_o t - \pi n)}{\omega_o t - \pi n}.$$

The individual contributions

$$C_n \frac{\sin(\omega_o t - \pi n)}{\omega_o t - \pi n} = C_n \frac{\sin \chi}{\chi} \quad \text{, say,}$$

where $\qquad C_n = f(\frac{\pi n}{\omega_o})$

and $\qquad \chi = \omega_o t - \pi n$

fall off as $1/\chi$ and are therefore localised to a small degree only, because $\frac{\sin\chi}{\chi} = O(\frac{1}{\chi})$ as $\chi \to \infty$.

It is possible to localise the individual contributions considerably more by the following procedure. We note that

$$\frac{\sin\chi}{\chi} = \sqrt{\frac{\pi}{2}} \frac{J_{\frac{1}{2}}(\chi)}{\chi^{\frac{1}{2}}} = O(\frac{1}{\chi})$$

and the general relation

$$\sqrt{\frac{\pi}{2}} \frac{2^n J_{n+\frac{1}{2}}(\chi)}{\chi^{n+\frac{1}{2}}} = \frac{1}{\pi n} (1+\frac{d^2}{d\chi^2})^n (\frac{\sin\chi}{\chi}) = O(\frac{1}{\chi^{n+1}}).$$

Hence, if instead of considering the function $f(t)$ itself, we construct the function

$$(1 + \frac{d^2}{d(\omega_o t)^2})^n .f(t)$$

we see that the individual contributions in the series expression for the above function are considerably more localised round the points $f(\frac{\pi n}{\omega_o})$ than is the case for the function $f(t)$ itself.

DR. J. J. GOOD.

I cannot help wondering whether it is not largely a prejudice to analyse signals in terms of frequency.

The prejudice arises partly because of the desire to ignore the apparatus which is to be used for interpreting the signal, $f(t)$. When the behaviour of the apparatus can be described by means of a differential equation of the form.

$$\frac{d^2\chi}{dt^2} + \omega^2\chi = f(t)$$

it is natural to think in terms of sine waves and frequencies. But this analysis is less appropriate when, for example, there is a term in $\frac{d\chi}{dt}$.

Similarly it may be that in quantum theory a lot of philosophical difficulties arise merely as a consequence of the insistance on analysing events in terms of frequency.

DR. D. K. C. MACDONALD

(1) The question of the ultimate "meaning" of the duality of frequency and time leads one to recall the very fundamental problem of the "meaning" of Planck's constant and its unit of measure - action. Although this may be expressed as energy x time or momentum x distance. etc., it still appears very difficult even yet for our brains to visualise although we have had Hamilton's Principle of Least Action available for so long as an analytic tool in dynamics.

(2) The importance of elementary $\frac{\sin t}{t}$ functions in this field (as in the "sampling theorem") calls to my mind some earlier essays in realisable network theory* where one finds that network having an amplitude-response $\sin f/f$ (and related derivatives) are inherently realisable. It appears to me now probable that the work could be extended to more general networks than seemed profitable at that time.

DR. D. GABOR (in reply)

Dr. Good has raised the interesting question whether the widespread use of Fourier analysis in communication problems has other grounds than just habit or prejudice? There are in fact two good reasons for this preference. In communication problems we deal usually with the infinite or semi-infinite time axis, moreover the problems are usually homogeneous in time. Once one or the other of these conditions is dropped, it may be well worth while to carry out the analysis in terms of other functions. For instance if the time interval considered is finite but homogeneous, gaussian elementary functions or functions of the type $\sin\omega t/\omega t$ may have advantages. If there is a distinguished instant of time, Hermite's orthogonal functions may be preferable. If both conditions are dropped one might choose Laguerre functions, Legendre functions Tchebysheff polynomials and a host of others. One can even argue that in communication problems in the narrower sense of the word there is always a distinguished instant, viz. the present, and in fact functions with a "perspectivic" view of the past may be the best.

Regarding the question posed by Dr. Macdonald, I do not think there is any problem in the ultimate "meaning" of the quality of frequency and time; they are simply dual by definition. The problem of the elementary action is quite another matter; it is one of the most fundamental of crude facts, and extremely refractory to visualization. It does not help much if one tries to "visualize" a momentum as the frequency conjugate to a co-ordinate. Dirac's remark is worth remembering: "The main object of physical science is not the provision of pictures but the formulation of laws governing phenomena".

* E.g. MacDonald, D. K. C. : Phil. Mag. 38, 115,(1947)